LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – NOVEMBER 2011

ST 1503 - PROBABILITY AND RANDOM VARIABLES

Date : 10-11-2011 Dept. No. Max. : 100 Marks

Time : 1:00 - 4:00

PART - A

Answer ALL the questions: ***(10 x 2 =20 Marks)***

1. Define Random Experiment.
2. What is the chance that a leap year selected at random will contain 53 Sundays?
3. State the Axiomatic Definition of Probability.
4. If $B \subset A$, show that $P(B) \le P(A)$.
5. Define Conditional Probability.
6. For two independent events A and B, P(A)= 0.4, P(B)= 0.5, Find P(A U B)
7. A bag contains 3 Red and 5 Green Balls. Two balls are drawn at random without replacement. Find the probability that both balls drawn are green.
8. Three percent of a given lot of manufactured parts are defective. What is the probability that in a sample of four items none will be defective?
9. A coin is tossed two times. Let X be the random variable denotes the number of Heads that occurred. Find the distribution of X and its mean value.
10. X and Y are independent variables with means 10 and 20 and variances 2 and 3 respectively. Find the variance of (3X + 4Y).

PART – B

Answer any FIVE Questions: ***(5 x 8 = 40 Marks)***

11. Five salesmen A,B,C,D,and E of a company are considered for a three member trade delegation to represent the company in an international trade conference. Construct the sample space and find the probability that (i) A is selected (ii) A is not selected and (iii) Either A or B (not Both) is selected.
12. An MBA applies for a job in two firms X and Y. The probability of his being selected in firm X is 0.7 and being rejected in Y is 0.5. The probability of at least one of his applications being rejected is 0.6. What is the probability that he will be selected by at least one firm?
13. For any three events A, B and C, prove that:
    $P(A \cup B \mid C) = P(A \mid C) + P(B \mid C) - P(A \cap B \mid C)$.
14. Let A and B be two events such that P(A) = 3 / 4 and P(B) = 5 / 8, show that:
    (i) $P(A \cup B) \geq 3/4$ and (ii) $3/8 \le P(A \cap B) \le 5/8$.
15. From a city population, the probability of selecting (i) a male or a smoker is 7 / 10. (ii) a male smoker is 2/5, and (iii) a male, if a smoker is already selected is 2 / 3. Find the probability of selecting (a) a non- smoker, (b) a male, and (c) a smoker, if a male is first selected.
16. The chances that doctor A will diagnose a disease X correctly is 60 % . The chances that a patient will Die by his treatment after correct diagnosis is 40 % and the chance of death by wrong diagnosis is 70%. A patient of doctor A, who had disease X, died. What is the chance that his disease was diagnosed correctly? **(P.T.O.)**

17. The incidence of a certain disease in an industry is such that on an average 20 % of workers suffer from it. If 7 workers are selected at random, what is the probability that 5 or more have got the disease? Also obtain the mean and standard deviation of the distribution.

18. Given the p.d.f of a continuous random variable X as follows:

$$f(x) = k\,x\,(1 - x), \quad 0 < x < 1$$
$$= 0, \quad \text{otherwise}$$

Find k, E(X) and Var(X).

PART - C

Answer any TWO questions: **(2 x 20 = 40 Marks)**

19. (a) A bag contains 4 red and 3 blue balls. Two drawings of 2 balls are made. Find the chance that the first drawing gives 2 red balls and the second drawing 2 blue balls
   (i) if the balls are returned to the bag after the first draw
   (ii) if the balls are not returned.

   (b) Three groups of children contain respectively 3 girls and 1 boy and 2 girls and 2 boys and 1 girl and 3 boys. One child is selected at random from each group.
   Find the chance that the 3 selected comprise 1 girl and 2 boys.

20. (a) Prove that if A and B are independent, then A^c and B^c are independent.
   (b) Show that $P(A \cap B) \geq P(A) + P(B) - 1$
   (c) A box contains 6 red, 4 white, 5 black balls. A person draws 4 balls from the box at random. Find the probability that among the balls drawn there is at least one ball of each colour. (6+6+8)

21 (a) State and prove Bayes Theorem.
   (b) A manufacturing firm products steel pipes in 3 factories with daily production of 500, 1000, and 2000 units respectively. According to past experience it is known that the fraction of defective outputs produced by the 3 factories are respectively 0.005, 0.008 and 0.01. A pipe is selected at random from a day's total production and found to be defective. What is the probability that the pipe came from the second factory?

22 (a) State and prove Addition Theorem for 3 events.
   (b) A thief has a bunch of 'n' keys. He tries the keys at random to rob a house. What is the probability that he will succeed in his r^{th} trial when he samples the keys (i) with replacement; (ii) without replacement

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